Design and Analysis of Adaptive Waveforms for Cognitive Radar

Vinayak Areguli, Vageesh Anand Dambal, Prasad L Hugar, M Uttara Kumari, B Roja Reddy
Dept. of ECE, RVCE, Bangalore

***Abstract* – The problem of radar waveform design has been a topic of research for last several decades. Since the inception of Radar technology, it has undergone plenty of improvements. One such considerable improvement is in the field of optimal waveform design to improve certain parameters of interest. In this paper, optimal waveform design techniques to maximize either Signal to Noise Ratio at the radar receiver or Mutual Information between the target impulse response and received signal are proposed. SNR and MI of each waveform are calculated for various scenarios by changing clutter, target and noise parameters. The designed waveforms are then analyzed using receiver operating curves to determine the probability of detection. Also, ambiguity diagrams are plotted to find out range and Doppler resolution of each designed optimal waveform. Peak to side lobe ratio is calculated to study the clutter rejection capability of the waveforms. Linear frequency modulated waveform is considered as a benchmark and results of designed waveforms are compared with it.**

Keywords – Cognitive Radar; Lagrangian multiplier; Power Spectral Density; Signal to noise ratio; Mutual information; Gaussian distribution; Ambiguity diagram; Peak to Side Lobe ratio.

I. INTRODUCTION

Much of the research effort devoted to radar signal processing in the literature has focused on optimizing the design of the receiver [1]. However, with advancements in the fields of digital signal processing, neural networks and machine learning, and optimization theory, and with the emergence of a new discipline called cognitive dynamic systems, the stage is set for an examination of the theory and design of cognitive radar systems, in which both the receiver and the transmitter are adapted to the environment [2]. Indeed, there is considerable evidence of such systems in nature, in the echolocation systems of bats and dolphins [3].

To establish the term cognitive radar, the notion of a cognitive cycle has to be understood. Figure 1 summarizes the essence of the cognitive cycle in its most basic form.
The key aspects are

• Perception of the environment.
• Control exercised on the environment by virtue of feedback of the information that was learnt through perception.

In light of this simplified view of cognition, the notion of a cognitive radar can be established as a complex dynamic system that

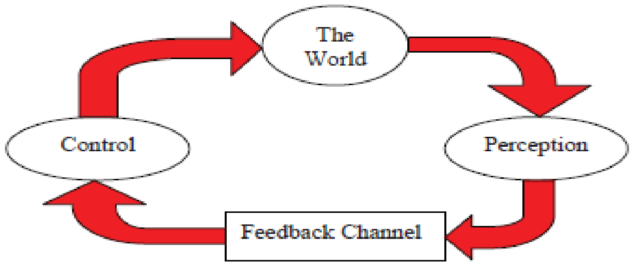

Figure 1: Basic form of Cognitive radar [2]

- continuously learns about the environment through experience gained from interactions with the environment, and updates the receiver with relevant information on the environment;
- adjusts transmitter's illumination of the environment in an effective and robust manner;
- Coordinates the operation of the transmitter and receiver using global feedback.

As suggested by the second item, the development of efficient algorithms for the design (or selection) of the transmitter's waveform is a key enabling step in the construction of a cognitive radar systems [5]. Such algorithms should provide a flexible framework that can synthesize waveforms that provide different trade-offs between a variety of performance objectives. The objectives themselves may also be adapted to the perceived nature of the environment.

Radar transmits electromagnetic energy in the form a waveform through its antenna, receives the echo and makes decisions about the target and the environment based on the echo signals. The echo signals are processed at the receiver using signal processing strategies. This can be thought of as imparting intelligence to the receiver.

General radar systems transmit wideband signals and chirp signals. These signals require large transmission power due to their large bandwidth. Extended targets have a narrow spectrum in most cases and hence only part of their spectrum needs to be excited to generate an echo. Therefore, using chirp signals results in wastage of transmission power and consequently reduces the Signal-to-Noise Ratio (SNR) at the output and is not recommended to be used as an optimal waveform.

To overcome this difficulty, if the transmitted waveform has to be designed according to the target and the environment, then it helps in better detection of target and subsequently reduces the number of false target detections (also called as false alarm).

This concept is central to Cognitive Radar which was first proposed by Haykin [2]. A CR updates its environment as soon as it is powered on as it becomes electromagnetically connected to the environment. A Cognitive radar observes and learns about its environment through a feedback system and adapts the transmitted waveform to improve system performance metrics such as probability of detection (P_d) and Constant False Alarm Rate (CFAR). It is used mostly in a resource constrained and interference concentrated environment [4].

The radar waveform design problem is an extremely challenging field since the radar engineer has to not only consider unknown and moving targets in unknown environments but also has to accept the fact that the end designed radar would cost up to a few hundred crores with a potential to provide security to an entire nation.

A. Problem Statement

Generally speaking, the approach to the design of an optimal radar waveform has been task-dependent. For example, for the task of detecting a particular target, the output signal to-noise ratio (SNR) should be maximized, and the optimal waveform puts all the available energy into the largest mode of the target. For the task of estimating the parameters of a target from a given ensemble, the radar waveform should distribute energy among different modes of the target in such a way as to maximize the mutual information between the received signal and the target ensemble [5].

It is of prime importance that the target be detected before any information from the target is extracted. To detect the target an appropriate waveform has to be transmitted with various constraints such as on the energy of the signal, duration of the signal, its repetition frequency and its modulus to name a few. Since the scattering phenomenon follows the super-position principle, a Linear Time Invariant Signal and System model is assumed.

The problem thus can be stated as:
Given the impulse response of the target h(t), it is required to design an optimal transmitted waveform x(t) which maximizes either SNR or MI, in the presence of clutter and noise.

Since a Cognitive Radar System is expected to work in situations where resources are available at a premium, certain constraints on the designed waveform exists. These are:

1. Energy of x(t) E_x (limited by antenna design).
2. Since radar waveforms have to be real, the signal x(t) must be time limited to [-T/2, T/2].

Based on the above considerations, waveforms are designed which conserve the bandwidth and hence help in efficient utilization of EM spectra along with providing better detection capabilities than conventional radar systems.

II. COGNITIVE RADAR

Figure 2 shows the cognitive cycle followed by a cognitive radar system. The cycle starts as the transmitter illuminating the environment. Here illumination refers to transmitter sending the signal into the surrounding environment. The radar returns generated by the environmental components such as target, and clutter are fed into two functional blocks at the radar receiver: radar-scene analyzer and signal processor. The signal processor makes decisions on the presence or absence of targets on continues time basis, using information on the environment provided to it by the radar-scene analyzer and the prior knowledge available in the radar system. The transmitter in turn illuminates the environment using the decisions made on possible targets, which are fed back to it by the receiver. During this feedback the waveform to be transmitted is optimized based on the performance requirements. This cycle is then repeated over and over again. Unlike a general communication system, the feedback mechanism which is a necessary requirement of a cognitive system is easy to implement as the radar transmitter and receiver are usually located at the same place. These types of radar where transmitter and receiver are at the same end are called as mono-static radar.

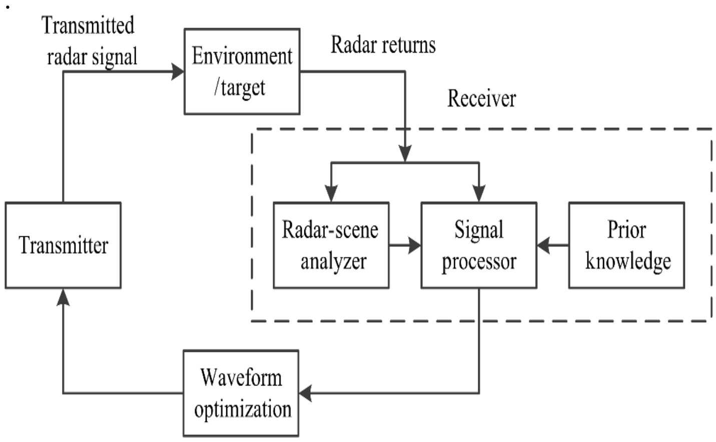

Figure 2: Cognitive radar as a closed loop feedback system

Based on Figure 2, a cognitive radar system consists of three **Major components:**

1. Intelligent signal processing block, which learns through the number of interaction between the radar and the potential targets and clutter.
2. A closed loop form of feedback from receiver to the transmitter which updates the intelligence of the transmitter.
3. Preservation of the information content extracted about the target and environment using the radar returns and making it prior knowledge for the future.

The transmitter of the cognitive radar selects its waveform based on the environmental status by using feedback structure from receiver to transmitter. Waveforms can be adaptively optimized based on the prior knowledge about the target and environment. The design and optimization of illumination waveforms to meet certain criteria is critical to the performance of a Cognitive radar system. The illumination signal must contain large enough bandwidth to excite the all resonant modes of the target in order to obtain a complete characterization of the target. Also, it must contain enough power in the dominant resonant modes to obtain useful target return signals.

III. METHODOLOGY

In order to achieve the objectives, the methodology is as follows:

1. Identification of the type of clutter (Gaussian or Swerling).
2. Determination of the clutter PSD.
3. Computation of the PSD of the extended target.
4. Calculation of ESD of required waveform subject to the maximization of MI and SNR under energy constraint.
5. Maximize the SNR and MI of the designed waveform with and without clutter and compared with the target spectrum follower (TSF) and linear frequency modulated waveform (LFM) for four scenarios.
6. Calculation of Signal to noise ratio and Mutual Information between target and received signal for all designed waveforms.
7. Analysis of Receiver operating curves to obtain the probability of detection of each designed optimal waveform.
8. Using the ambiguity diagram for each waveform to determine their Doppler resolution and range resolution capabilities.
9. Optimization of Polyphase waveform to improve SNR and MI.
10. Determination of Peak to side lobe ratio of each waveform to study its clutter rejection capability.

Let x(t) be a finite energy signal with duration of T which has a bandwidth of w. The energy of the signal is given by

$$\mathrm{Ex} = \int_{-T/2}^{T/2} |\mathrm{X(t)}|^2 \, \mathrm{dt}$$

Clutter PSD is given by the Gaussian distribution

$$\mathrm{P}_c(f) = \mathrm{g_c} \exp\left(-\frac{(\mathrm{f}-\mathrm{f_c})^2}{2\,\sigma_c^2}\right)$$

Extended target is assumed to have Gaussian mixture shape with PSD given by

$$|\mathrm{H(f)}|^2 = \mathrm{a_0} + \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{a_i} \exp\left(-\frac{(\mathrm{f}-\mathrm{f_i})^2}{2\,\sigma_T^2}\right)$$

The Lagrangian multiplier optimization is used to find out the PSD of transmitted waveform to maximize SNR and MI with constraint on its energy.
The waveform designed to maximize MI is given by

$$|\mathrm{X(f)}|^2 = \mathrm{Max}\left[0, \frac{\left(\frac{|\mathrm{H(f)}|^2}{\lambda\,\mathrm{P_n(f)}} - 1\right)}{\frac{|\mathrm{H(f)}|^2}{\mathrm{P_n(f)}}}\right]$$

Similarly, the PSD of the waveform which maximizes SNR is

$$|X(f)|^2 = Max\left[0, \left(|H(f)|\sqrt{\left(\frac{P_n(f)}{\lambda}\right)} - P_n(f)\right) / \, P_c(f)\right]$$

IV. RESULTS OF THE DESIGNED WAVEFORMS

The adaptive waveform design has been carried out for four different scenarios by varying clutter, target and noise in each scenario. In each scenario SNR, CMI, MI, TSF and LFM based waveforms are analyzed. For simplicity we will discuss the results in Gaussian and Swerling clutter for one scenario.

A. Gaussian clutter

In scenario 1 extended target is modeled using Gaussian mixture model. Clutter is modeled as Gaussian distribution. Four waveforms are studied in each scenario. For each waveform ambiguity function and corresponding graphs are generated and results are analyzed.

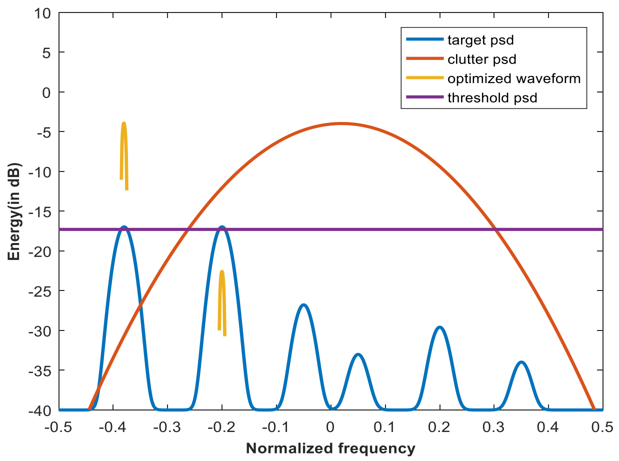

Figure 3: PSD of the designed optimal waveform along with target, clutter and noise psd

In figure 3, PSD of the optimum waveform which maximizes SNR is plotted. The threshold PSD is obtained by Lagrangian multiplier technique as described in chapter 3. Whenever target PSD is more than this threshold transmitter puts energy in those frequency bands. Since this waveform is aimed at maximizing SNR it puts greater amount of power in the band [-0.4, -0.38] where target PSD is higher than clutter PSD. By putting more transmitter power in largest mode of the target SINR is maximized.

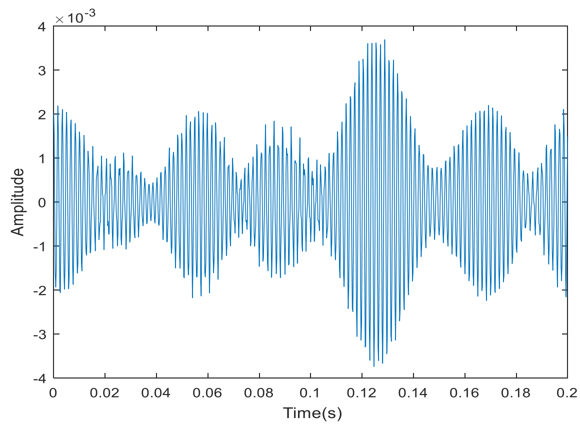

Figure 4: CSNR based Time domain waveform

Figure 4 shows the time domain waveform of the SNR based waveform. Time domain waveform is obtained as follows. First step is to take the square root of PSD which gives the magnitude of Fourier transform of the signal. Then random phase is introduced to the obtained magnitude. By taking the inverse Fourier transform of this, we get the time domain waveform. Since many time domain waveforms have the same PSD, waveform obtained is not unique.

To identify the target, it is best to maximize the SNR at the receiver. Hence from table 1 it can be seen that the CSNR based waveform has the maximum SNR in each scenario. Hence to detect the target the CSNR based waveform has to be transmitted. On the other hand, to gain information from the target it is required to maximize the Mutual Information between the target echo and received signal. Hence to achieve this waveform designed to maximize the MI has to be transmitted. From the table it can be observed that the CMI based waveform is suitable for such a scenario.

Table 1: SNR and MI values of designed waveforms

	MI	CMI	CSNR	TSF	LFM	Polyphase
SNR Scenario 1	16.9580	17.7846	**18.0170**	15.6841	7.2902	17.7847
MI	16.5731	**17.2191**	16.7158	15.5504	7.2790	14.0000
SNR Scenario 2	17.9934	17.989	18.2308	16.1486	7.3815	**20.567**
MI	17.5331	17.5341	14.9657	15.9998	7.3703	**17.5580**
SNR Scenario 3	19.6901	22.3085	**22.7864**	19.5468	12.5627	22.2259
MI	19.1498	**21.189**	20.0351	19.2608	12.5269	16.6542
SNR Scenario 4	25.0861	26.6902	**27.2646**	24.1624	16.7358	26.3005
MI	23.6855	**25.6826**	23.9243	23.2239	16.6330	21.0526

B. Swerling clutter

Swerling distribution is used to model moving objects. With this clutter model SNR, CMI and MI based waveforms are analyzed.

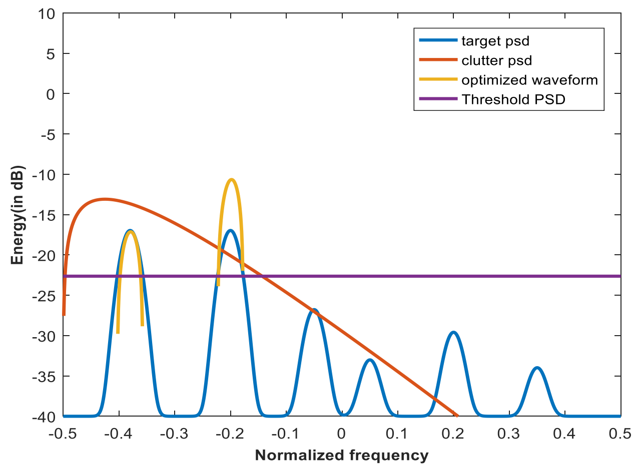

Figure 5: PSD of the designed waveform in Swerling clutter

Similar to Gaussian clutter case here also waveform puts energy in those bands where target PSD is higher and clutter PSD is lower. In the figure 5 more energy is allocated in the frequency band around -0.2.

V. CONCLUSION AND FUTURE SCOPE

In this work waveforms have been designed for 4 different scenarios wherein the distinguishing factors are the target model, clutter model and interference and noise level. The target is modelled as a deterministic extended target and clutter is assumed to have a Gaussian PSD. Waveforms are designed based on energy allocation constraints using the Lagrangian Multiplier technique and maximizing SNR and MI in situations with and without clutter for the MI and SNR case which are called CMI and CSNR respectively. It is also found that designing waveform based on TSF method is not optimum and hence energy must be allocated to only those bands of the target modes where target power is maximum and clutter power is the least and not at all peaks of the target. The performance of these waveforms characterized by their probability of detection is analysed to be better than conventional chirp and wideband signal. For all the waveforms designed the ROC plot is used to determine the probability of false alarm and probability of detection. The characteristics of these waveforms are determined by plotting their ambiguity functions to determine their performance based on range and resolution. The ISL and PSL of these waveforms have also been determined to find their clutter rejection abilities.

This wok can be extended to determine the performance of these waveforms for stochastic targets using Kalman Filter based iterative algorithms. Also, this design procedure can be easily extended to the case of MIMO radars, wherein the number of receiver implementations has to be increased. The above work can also be analysed in non-Gaussian clutter. This is an acceptable assumption as new high-resolution radars are being developed. As the radar resolution increases, the clutter shows non-Gaussian characteristics.

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Mr. Vinayak Areguli completed his BE from R. V. College of Engineering, Bangalore in Electronics and Communication Engineering. His research interests are in Radar Signal Processing, Very Large-Scale Integration, Digital System Design and Analog Microelectronics. He is currently working in a Wireless Technology Industry in Bangalore.

Mr. Vageesh Anand Dambal completed his BE from RV College of Engineering, Bangalore in Electronics and Communication Engineering. His research interests are in Signal Processing, Computational Intelligence and Communication Systems. He is currently pursuing his MS in Electrical Engineering at North Carolina State University, Raleigh.

Mr. Prasad Hugar completed his BE from R. V. College of Engineering, Bangalore in Electronics and Communication Engineering. His research interests are Analog Microelectronics, Signal Processing, Very Large-Scale Integration Design and Communication Systems. He is currently working in a Semiconductor Industry in Bangalore.

Dr. Uttara Kumari M received her B. E degree in 1989 from Nagarjuna University, Hyderabad, Andhra Pradesh, ME degree in 1996 from Bangalore University, Karnataka and Ph.D degree in 2007 from Andhra University. She is presently working at R V College of Engineering for the past 20 years. Her research interests lie in the areas of Radar Systems, Space Time Adaptive Processing, Speech & Image processing and Sensors.

Dr. Roja Reddy B received the B. E degree in 1998 from Gulbarga University, Karnataka, M E degree in 2004 from VTU, Karnataka and Ph. D degree in 2016 from VTU, Karnataka. She is presently working at R V College of Engineering with an experience of 13 years in the field of teaching. Her research interest lies in the areas of Radar Signal Processing.